RIGOROUS AND EFFICIENT ANALYSIS OF HYBRID T-JUNCTIONS

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Abstract

Although hybrid T-junctions (magic T's) are widely used in common microwave practice, no efficient method of analysis of such junctions is at present available. In this contribution we propose a very simple, yet rigorous and efficient model for the full-wave characterization of these junctions. The model is based on a suitable segmentation of the structure in conjunction with the admittance matrix description. When suitable modal expansions of the Green's functions are considered, a very high numerical efficiency is obtained. Numerical simulations show excellent agreement with measured data.

1. Introduction

Multiport junctions are frequently used to achieve the desired power division among different waveguides. In particular, T-junctions, both in the E and H planes (later referred to as planar T-junctions) and magic-T junctions (which will be referred to as 3-D T-junctions) are among the most commonly employed. The analysis of planar T-junctions has been addressed in many papers [1-3]. In [1] a very simple and efficient method to describe the planar junction by means of the admittance matrix representation has been introduced. Recently, an approach based on the three plane mode-matching has been followed in [2]. In such an approach the conventional mode-matching technique, based on the characterization of the individual discontinuities by means of the generalized scattering matrix, has been used. In order to maintain the boundary value problem tractable, one of the three ports was consecutively short circuited and the resulting structure analyzed. As a consequence, three different analyses of the double step discontinuity have to be performed. Finally, in [3] modal analysis has been used to describe several planar junctions with the scattering superposition technique.

The 3-D magic T-junction has received considerably less attention. In this case the conventional mode-matching technique, although feasible, leads to codes which are both involved and poorly efficient. On the contrary, the 3-D T-junction can be treated in a straightforward manner by applying the segmentation technique introduced in [4, 5] for the analysis of branch-guide coupler. The same technique can be used both for the simpler planar T-junction as well as for more complicated multiport structures (designed to fulfill special requirements of power/phase distribution among different ports), which can thus be analyzed in an extremely easy and efficient way.

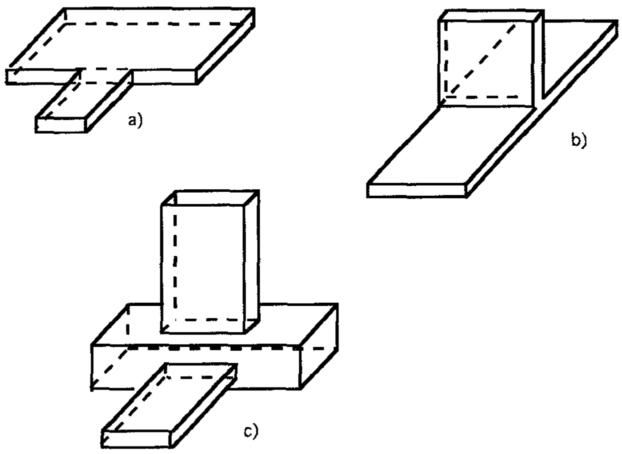

Fig. 1 . a) H-plane T-junction; b) E-plane T-junction; c) hybrid T-junction (magic T)

A 3-D T-junction is sketched in Fig. 1c. Fig. 2 shows the same structure with the feeding waveguides removed. This is just a rectangular resonator with four apertures on its walls. The analysis can be accomplished very easily using the admittance matrix representation. It is sufficient to compute the magnetic field on the various apertures

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when these are short-circuited, i.e. when the resonator is completely closed (with apertures replaced by equivalent magnetic currents). As a consequence, the involved boundary value problem is greatly simplified. With this technique one does not need to analyze discontinuities, but just to characterize the very simple geometry of the resonator. Moreover, this characterization leads to a straightforward network description of the overall discontinuity (3-D junction), thus providing further insight into the coupling mechanism between the various ports.

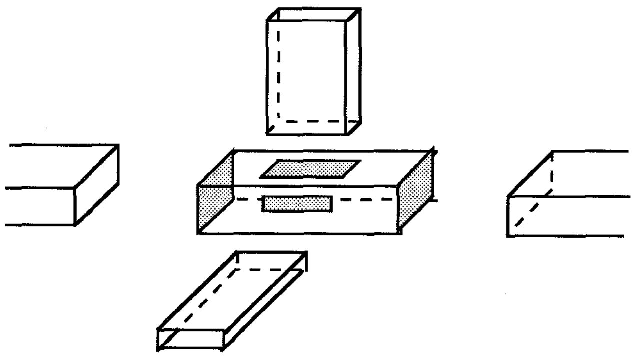


Fig. 2. Segmentation of the hybrid Tee into a resonator R_o and four feeding waveguides. Shadowed regions correspond to the apertures on the resonator

2. Theory

Let R_o be the resonator of Fig. 2, where four physical outputs S_i (i=1,..,4) to be connected to the four waveguides are produced. The EM field in R_o is determined uniquely by the knowledge of the tangential electric field over the output S_i. By slightly modifying the treatment in [6], the magnetic field in R_o can be expressed in general as

$$\mathbf{H}(\mathbf{r}) = j\omega\varepsilon \oint_S \mathbf{n}\times\mathbf{E}(\mathbf{r}')\, \underline{\underline{\mathbf{G}}}(\mathbf{r}, \mathbf{r}')\, d\mathbf{S}' =$$

$$j\omega\varepsilon \sum_{i=1}^{N_p} \int_{S_i} \mathbf{n}\times\mathbf{E}(\mathbf{r}')\, \underline{\underline{\mathbf{G}}}(\mathbf{r}, \mathbf{r}')\, d\mathbf{S}' \qquad (1)$$

the integral over S being reduced to the only portions S_i where $\mathbf{n}\times\mathbf{E}$ is not zero. In (1) the quantity $\mathbf{n}\times\mathbf{E}$ can be interpreted as a magnetic surface current flowing on S_i. $\underline{\underline{\mathbf{G}}}$ is the dyadic Green's function for the magnetic field in R_o. It is a solution of

$$\nabla\times\nabla\times\underline{\underline{\mathbf{G}}} - k^2\underline{\underline{\mathbf{G}}} = \underline{\underline{\mathbf{I}}}\, \delta(\mathbf{r}-\mathbf{r}') \qquad \text{in } R_o \qquad (2)$$

$$\mathbf{n}\times\nabla\times\underline{\underline{\mathbf{G}}} = 0 \qquad \text{on } S \qquad (2')$$

where $k^2 = \omega^2\mu\varepsilon$, $\underline{\underline{\mathbf{I}}}$ is the unit dyadic, δ is the Dirac delta function.

The relationships among field quantities at the openings S_i can be formulated in terms of a generalized admittance matrix by expanding the tangential E- and H-fields into the modes of the feeding waveguides,

$$\mathbf{n}\times\mathbf{E}^{(i)} = \sum_{k=1}^{N_i} V_k^{(i)} \phi_k^{(i)} \qquad (3a)$$

$$\mathbf{H}^{(i)} = \sum_{k=1}^{N_i} I_k^{(i)} \phi_k^{(i)} \qquad (3b)$$

N_i being the number of waveguide modes taken into account on S_i. The series in (3a,b) are truncated to N_i terms for numerical reasons. The expansion coefficient $V_k^{(i)}$ and $I_k^{(i)}$ represent the equivalent voltage and current on the *kth* electrical port on the output S_i. They are related to the respective field quantities on S_i by

$$V_k^{(i)} = \int_{S_i} \mathbf{n}\times\mathbf{E}(\mathbf{r})\, \phi_k^{(i)}(\mathbf{r})\, d\mathbf{r} \qquad (4a)$$

$$I_k^{(i)} = \int_{S_i} \mathbf{H}(\mathbf{r})\, \phi_k^{(i)}(\mathbf{r})\, d\mathbf{r} \qquad (4b)$$

Combining (4b) with (1) and (3a) we obtain

$$I_m^{(j)} = \sum_{i=1}^{N_p}\sum_{k=1}^{N_i} Y_{mk}^{(ji)} V_k^{(i)} \qquad (5)$$

where

$$Y_{mk}^{(ji)} = \int_{S_j}\int_{S_i} \phi_m^{(j)}(\mathbf{r}) \cdot \underline{\underline{\mathbf{G}}}(\mathbf{r},\mathbf{r}') \cdot \phi_k^{(i)}(\mathbf{r}')\, d\mathbf{r}\, d\mathbf{r}' \qquad (6)$$

defines the generalized admittance matrix of R_o. This quantity represents the amplitude of the *mth* current component entering the *jth* opening produced by a unit *kth* component of the voltage at the *ith* opening, all the other voltage components being zero.

Observe that the same method can be used no matter how many waveguides are connected to R_o. It therefore can be applied to a stub structure, to an ordinary T-junction (E- or H-plane) as well as to a cross-junctions, etc. It is also noted that no matrix inversion is necessary to compute the generalized admittance matrix. This is due to the fact that the admittance matrix is the natural representation of a cavity with conducting walls.

3. Results

The theory developed in the previous sections enables us to calculate in a very effective way the admittance and then the scattering matrix, of a hybrid T-junction (magic T). In order to validate the theory, we have compared our results with previuosly published data. In particular, as a first example, planar T-junctions have been simulated and fig. 3 shows the S-parameters obtained for an H-plane waveguide T-junction simulated and measured by Liang et al. in [2]. A good agreement is observed between both theories.

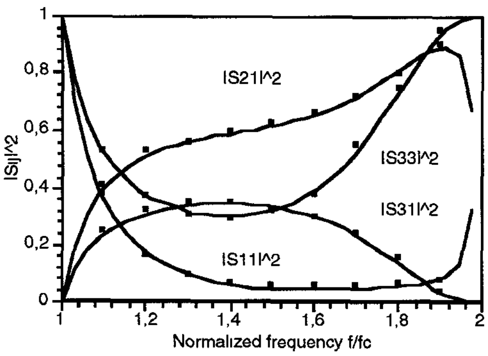


Fig. 3 Scattering parameters of the H-plane T-junction withWR-90. The dots refer to the simulations of [2].

A further comparison is provided in Fig 4 where the E-plane T junctions analyzed by Sieverding and Arndt [3] are compared with our results. Also in this case, a very good agreement can be observed.

Finally, Fig 5a shows the results of the theoretical simulations for a magic T-junction made with four identical WR62 waveguides, while the relative geometry as well as the port numbering is depicted in fig. 5b.

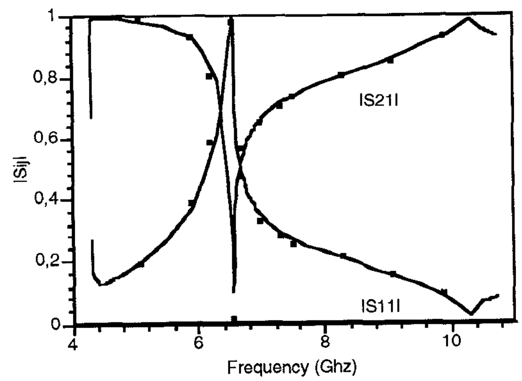
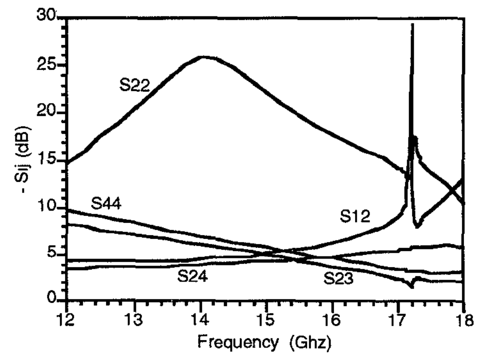


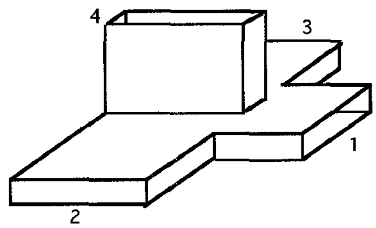
Fig. 4 .E-plane T-junction with unequal width waveguides (port 1, 2 refer to a WR-137, while port 3 refers to a WR-90). The dots represent the theoretical computations reported in [1, 3]

4 Conclusions

Although hybrid T-junctions (magic T) are widely used in common microwave practice, few methods of analysis of such junctions are at the present available. In this contribution, by using a suitable segmentation of the structure and the admittance matrix description, we have introduced a rigorous and numerically very efficient technique for the full-wave characterization of the hybrid T-junction discontinuity. Numerical simulations have been compared with measured data confirming the high degree of accuracy attained.



a)



b)

Fig. 5. Scattering parameters (a) and geometry and port numbering (b) of the hybrid Tee.The waveguide used is the WR-62.

References

[1] E. D. Sharp, "An exact calculation for a T-junction of rectangular waveguides having arbitrary cross-sections," *IEEE Trans. Microwave Theory Tech*, vol. MTT-15, pp. 109-116, Feb.1967.

[2] X-P. Liang, K. Zaki, and A. E. Atia, "A rigorous three plane mode-matching technique for characterizing waveguide T-junctions, and its application in multiplexer design," *IEEE Trans. Microwave Theory Tech,* vol. MTT-39, pp. 2138-2147, Dec.1991.

[3] T. Sieverding, and F. Arndt, "Field Theoretic CAD of open or aperture matched T-junction coupled rectangular waveguide structures," *IEEE Trans. Microwave Theory Tech*, vol. MTT-40, pp. 353-361, Feb.1992.

[4] F. Alessandri, M. Mongiardo, R. Sorrentino, "Transverse segmentation: a novel technique for the efficient CAD of 2N-port branch-guide couplers," *IEEE Microwave and Guided Wave Letters*, vol. 1, No. 8, Aug. 1991.

[5] F. Alessandri, M. Mongiardo, R. Sorrentino, "A technique for the full-wave automatic synthesis of waveguide components: application to fixed phase shifters," *IEEE Trans. Microwave Theory Tech*, vol. MTT-40, No. 7, July 1992.

[6] R. E. Collin, *Field Theory of Guided Waves*, IEEE Press, New York 1991, Sec. 2.18